STATE OF CALIFORNIA
DEPARTMENT OF PUBLIC WORKS
DIVISION OF HIGHWAYS

A Report on

EVALUATING THE UNIFORMITY of PORTLAND CEMENT CONCRETE

57-22

MATERIALS & RESEARCH DEPARTMENT

State of California
Department of Public Works
Division of Highways
MATERIALS AND RESEARCH DEPARTMENT
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April 15, 1957

Mr. J. W. Trask
Assistant State Highway Engineer
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Sacramento, California

Dear Mr. Trask:

Submitted for your consideration is:

A report on

EVALUATING THE UNIFORMITY
of
PORTLAND CEMENT CONCRETE

Study made by	Technical Section
Under general direction of	Bailey Tremper
Report Prepared by	W. E. Haskell

Yours very truly

F. N. Hveem

F. N. Hveem
Materials & Research Engr.

EVALUATING THE UNIFORMITY OF PORTLAND CEMENT CONCRETE

One of the professed objectives in every scheme for the design and control of portland cement concrete, is that of obtaining a product of "uniform" quality. It is interesting to note however, that while uniformity has been talked about from time immemorial, it is only recently that any official criteria for evaluating this attribute have been proposed. The criteria referred to were published in 1955 by A.C.I. Committee 214[1], and the basis for the evaluation of the uniformity of the concrete, is the coefficient of variation of the compressive strength test results.

This report is for the purpose of providing information on the findings of the committee and for showing the results obtained in the examination of some data in the possession of the Materials and Research Department. Other statistical measures not considered by the committee are also briefly discussed.

(1) A.C.I. Committee 214, "Evaluation of Compressive Test Results of Field Concrete" J. Amer. Concrete Inst. No. 3, Vol. 27, November, 1955

The Computation of Statistical Measures

The coefficient of variation is the standard deviation of a set of observations, expressed as a percentage of the arithmetic mean or average value of the set. The standard deviation is the root-mean-square deviation of the values from their average. The definitive expression of this function is,

$$\sigma = \sqrt{\frac{(X_1-\bar{X})^2 + (X_2-\bar{X})^2 + \cdots + (X_n-\bar{X})^2}{n}} \qquad \ldots\ldots.(1)$$

Where σ = the standard deviation
$X_1 \cdots X_n$ = observed values of a measurable characteristic
$\bar{X}$ = the arithmetic mean or average of a set of observed values
n = the number of observed values

For facility in computation, this expression is frequently rearranged as

$$\sigma = \sqrt{\frac{X_1^2 + X_2^2 + X_3^2 + \cdots + X_n^2}{n} - \bar{X}^2} \qquad \ldots\ldots.(2)$$

The coefficient of variation (v) is,

$$v = \frac{\sigma}{\bar{X}} 100 \qquad \ldots\ldots.(3)$$

Where more than one test specimen is fabricated from each batch sample, and it is desired to compute the batch-to-batch standard deviation, the expression becomes,

$$\sigma = \sqrt{\frac{\bar{X}_1^2 + \bar{X}_2^2 + \bar{X}_3^2 + \cdots + \bar{X}_n}{n} - \bar{\bar{X}}^2} \qquad \ldots\ldots.(4)$$

and the batch-to-batch coefficient of variation is,

$$v_2 = \frac{\sigma}{\bar{\bar{X}}} 100 \qquad \ldots\ldots.(5)$$

Where, $\bar{X}_1 \ldots \bar{X}_n$ = Averages of companion specimens from each batch

$\bar{\bar{X}}$ = Average of the averages

v_2 = Batch-to-batch coefficient of variation

The symbols used in the above equations are those favored by the ACI Committee, and also by the American Society for Testing Materials(2), and for that reason are used in this report. In most modern statistical texts however, a distinction is made between the symbols for the universe, and sample statistics. For example, the symbol "σ" is used to denote the standard deviation of a universe that is of a hypothetical set of observations consisting of an infinite number of individual items. The symbol "s" is used to denote the standard deviation of a sample which must of necessity, be of finite size. In order that "s" shall be an unbiased estimate of σ, the denominator used in the equation for computing s is n-1 instead of n. If the number of items in a sample is 30 or more, the difference between the two equations becomes unimportant.

A small coefficient of variation denotes a uniform concrete; a large one is a sign of non-uniformity. Table I, taken from the committee report is a record of proposed standards in terms of the coefficient of variation.

TABLE I
Standards of Concrete Control

Class of Operation	Coefficients of Variation for Different Control Standards			
	Excellent	Good	Fair	Poor
Over-all variations General Construction	Below 10.0	10.0 to 15.0	15.0 to 20.0	Above 20.0
Laboratory Control	Below 5.0	5.0 to 7.0	7.0 to 10.0	Above 10.0
Within-batch Variations, Field Control	Below 4.0	4.0 to 5.0	5.0 to 6.0	Above 6.0
Laboratory Control	Below 3.0	3.0 to 4.0	4.0 to 5.0	Above 5.0

The Materials and Research Department of the California Division of Highways began the statistical study of job concrete a number of years prior to the publication of the committee report. Table II is a record of the comparison of the uniformity of the concrete from fifteen representative projects.

(2) "ASTM Manual on Quality Control of Materials", Special Technical Publication No. 15-C (January, 1951)

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TABLE II
A Comparison of the Uniformity of Portland Cement Concrete from Fifteen Pavement and Structural Projects

Project Number	Number of Test Specimens	Mean Compr. Strength at 28 days psi	Standard Deviation psi	Coefficient of Variation, Per Cent
Pavement Concrete				
1	47	4375	445	10.2
2	33	3425	372	10.9
3	41	5325	565	10.6
4	42	3380	430	12.7
Structural Concrete				
5	916	4657	457	9.8
6	361	4345	573	13.2
7	240	4451	476	10.7
8	115	3803	460	12.1
9	67	3812	404	10.6
10	50	4320	480	11.1
11	47	3885	597	15.4
12	45	4010	418	10.4
13	50	4590	440	9.6
14	40	4450	808	18.2
15	39	4600	823	17.9

A comparison of the coefficients of variation in the above table with the criteria proposed in Table I, indicates that the uniformity of the concrete in a majority of the projects is good or excellent.

In the projects listed above, each test specimen was from a separate batch of concrete, and in general, only one specimen was taken for each day's pour. In a series of tests of this kind, it is impossible to arrive at any estimate with regard to the precision of the work of the operators and the equipment used in making the tests. Such an estimate can easily be made however, if two or more companion specimens are made from each test sample. The method of making such an estimate will be illustrated by an example. As a matter of fact, thirty or more replicate sets should be obtained where possible, but in the example only ten such sets will be examined. Table III is a record of these tests.

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TABLE III
Evaluation of Testing Procedures

Sample	28 Day Strength, psi		Range
	Specimen No. 1	Specimen No. 2	
1	4150	3960	190
2	3220	3070	150
3	3250	2980	270
4	3530	3630	100
5	3210	2940	270
6	3710	3520	190
7	4000	4020	20
8	2850	3050	200
9	3450	3620	170
10	4060	4150	90

Average Compressive Strength, psi	$\bar{X}$ = 3518
Average Range, psi	$\bar{R}$ = 165
Standard Deviation, psi	σ = 423
Coefficient of Variation, per cent	v = 12

The within-batch standard deviation, which in this example is a measure of the precision of the testing procedure, may be very easily computed by the use of the expression

$$\sigma_1 = \bar{R}\frac{1}{d_2} \qquad \text{....... (6)}$$

Where,

σ_1 = The within-batch standard deviation
$\bar{R}$ = The average range
$\frac{1}{d_2}$ = A factor, which depends upon the number of test specimens in each sample

The values of d_2 are given in numerous statistical texts, and in the ASTM Manual referred to previously(2). Table IV is an abbreviated table of d2 and its reciprocal.

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TABLE IV

Number of Observation in the Sample	d_2	$\frac{1}{d_2}$
2	1.128	0.8865
3	1.693	0.5907
4	2.059	0.4857
5	2.326	0.4299

By using the above expression, and the constants listed in Table IV, the value of σ_1 becomes an unbiased estimate of the within-batch universe standard deviation.

The within-batch standard deviation computed from the data in Table III is σ_1=165x0.8865=146 psi and the corresponding coefficient of variation is $\frac{146}{3518}$ x 100 = 4.15 per cent.

A comparison of this value with the standards given in Table I indicates that the precision of the testing procedure is good.

The over-all coefficient of variation of the data given in Table III is 12 per cent and as we have just seen above, the coefficient of variation attributable to the testing procedures is in round figures, 4 per cent. With this information available, it is possible to compute the coefficient of variation due to job practices by themselves, using the expression

$$v_j = \sqrt{(v_o)^2 - (v_t)^2}$$

Where v_j = job coefficient of variation
v_o = overall coefficient of variation
v_t = variation due to testing procedures

In the above example, the job coefficient of variation is

$$v_j = \sqrt{(12)^2 = (4)^2} = 11.3 \text{ per cent}$$

Statistical Procedures in the Design of Concrete Mixtures

From the foregoing, it is evident that appreciable variations in compressive strength will be encountered whenever any considerable number of tests are made over an extended period, even though the overall coefficient of variation can be classified as "good." This is easily seen in the data of Table III. The average compressive strength is 3518 psi, the overall standard deviation is 423 psi and the coefficient of variation is 12 per cent. We may assume for our example that this data is representative of what a certain individual ready-mix concrete manufacturer can do.

It is apparent that if this manufacturer expects to furnish concrete that will not show any test results, or only a small fraction of the test results below 3500 psi, he will have to furnish concrete with an appreciably higher average strength. Just how much higher in strength this concrete should be can be readily estimated.

In the committee report, it is stated, "As a general guide, it is the opinion of the committee that a reasonable control of structural concrete would be provided if no more than one test in ten fell below the value of f c' used in design. This tolerance of test failure does not imply acceptance of consecutive failures in 10 per cent of the structures but must be expressed as a continuous control rather than an over-all percentage. Additional low strength specimens are allowable in general concrete construction but the final criterion adopted is obviously a matter for the designer's decision based on his intimate knowledge of the conditions that are likely to prevail.

"To satisfy strength performance requirements expressed in this fashion the average strength of concrete must obviously be in excess of f c', the degree of excess strength depending on the expected uniformity of concrete production and the allowable proportion of low tests. The required average strength of f c r can be approximated as follows:

$$f\,c\,r = \frac{f\,c'}{(1 - t\,v)} \qquad \ldots\ldots\ldots(7)$$

where f c r = required average strength
f c' = design strength specified
t = A constant, depending upon the proportion of tests that may fall below f c' and the number of samples used to establish v.
v = The forecasted value of the coefficient of variation."

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Table V, taken from the committee's report, is used to obtain the t values. Those familiar with statistics will recognize this table as a modified arrangement of the well-known "student's t" tables, which are published in all statistical texts. The principle difference in Table V is that the column headings are given as percentages (99, 98, 95, 90, etc.) whereas in the usual t tables, they are given as the proportions (0.01, 0.02, 0.05, 0.10, etc.). When designated as in Table V, they are called "confidence levels." When designated as in the usual t table, they are termed "significance levels."

In Table III, there were 10 duplicate samples used to establish the value of v, and to meet the criterion of not more than 1 in 10 falling below the lower limit, the value of t to be used in the computations is found in Table V in the ninth row and the fifth column. It is 1.383.

TABLE V
Values of t

No. of Samples Minus 1*	Percentage of tests falling within the limits X ± t σ							
	50	60	70	80	90	95	98	99
	Chances of falling below lower limit							
	1 in 4	1 in 5	1 in 6.7	1 in 10	1 in 20	1 in 40	1 in 100	1 in 200
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576

*Degrees of freedom
Values of t extracted from table originally produced by Fisher and Yates, "Statistical Tables for Biological Agriculture and Medical Research."

Table continued on next page

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Table V
(Continued)

Other values of t for $n - 1 = \infty$

Percentage Within $X \pm t\sigma$	Chances of falling below lower limit	t
33.33	1 in 3	0.431
68.27	1 in 6.3	1.000
95.45	1 in 44	2.000
99.73	1 in 741	3.000

The required average strength is

$$f\ c\ r = \frac{3500}{\left[1-(1.383 \times 0.12)\right]} = 4197 \text{ psi}$$

in accordance with the committee's recommendations.

If low strength results were not considered to be critical, and two low tests in ten could be tolerated, the concrete manufacturer could aim for an average strength of

$$f\ c\ r = \frac{(3500 \times 0.9)}{\left[(1-(1.383 \times 0.12)\right]} = 3777 \text{ psi}$$

The above computations indicate rather clearly that a concrete manufacturer who is operating with a high coefficient of variation is at a considerable economic disadvantage when called upon to produce concrete of predetermined strength.

Confidence Limits of the Average

In nearly every case where a number of results of compressive strength tests are examined, their arithmetic mean or average is computed. This statistic is an unbiased estimate of the universe average - that is, it is the best <u>single estimate</u> of the universe average that can be made. It is probable that in many instances it is believed that this computed average is closer to the true universe average than is actually the case. It is possible to estimate the universe average in another way, that is, by calculating the limits within which the true average will be likely to be found at a given probability level. These limits are called "confidence limits" and are easily computed.

The standard deviation or standard error <u>of the mean</u> is computed as follows:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \qquad \ldots\ldots\ldots (8)$$

Where $\sigma_{\bar{x}}$ is the standard error of the mean and the other symbols have their usual meaning. It is expressly understood that the value of σ has been computed from 30 or more items of data if equation (8) is to be used.

An example of the computation of confidence limits can be given by using the data of the second project under structural concrete in Table II. The numerical values necessary in the computation are:

$n = 361$ $\qquad\qquad$ $\bar{X} = 4345$ psi

$\sigma = 573$ psi $\qquad\qquad$ $\sqrt{361}$ - 19 psi

The standard error of the mean is computed as

$$\sigma_{\bar{x}} = \frac{573}{19} = 30.2 \text{ psi}$$

Referring to Table V under ∞, the t values for probabilities of 95 and 99 percent are found to be 1.960 and 2.576. Using these values, the confidence limits are computed as follows:

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$$l_1 = \overline{X} - (\sigma_{\overline{X}} \times t)$$
$$l_2 = \overline{X} + (\sigma_{\overline{X}} \times t) \quad \ldots\ldots\ldots (9)$$

Using the numerical values given in the preceding paragraphs, the confidence limits are computed as follows:

$$l_1 = 4345 - (30.2 \times 1.960) = 4286$$
$$l_2 = 4345 + (30.2 \times 1.960) = 4404$$

$$l_1 = 4345 - (30.2 \times 2.576) = 4267$$
$$l_2 = 4345 + (30.2 \times 2.576) = 4423$$

Table VI is a record of the confidence limits of the averages of the projects listed in Table II.

TABLE VI
Confidence Limits of Average Compressive Strength

Proj. No.	Average Compr. Strength psi	Confidence Limits			
		Probability 0.95		Probability 0.99	
		Upper	Lower	Upper	Lower
1	4375	4506	4244	4550	4200
2	3425	3558	3292	3604	3246
3	5325	5504	5146	5563	5087
4	3380	3513	3247	3558	3202
5	4657	4686	4628	4696	4618
6	4345	4404	4286	4423	4267
7	4451	4512	4390	4532	4370
8	3803	3888	3718	3915	3691
9	3812	4910	3714	3942	3682
10	4320	4457	4183	4502	4138
11	3885	4060	3710	4119	3651
12	4010	4135	3885	4177	3843
13	4590	4714	4466	4756	4424
14	4450	4709	4191	4796	4104
15	4600	4867	4333	4957	4243

As might also be expected, it is also possible to estimate how many test samples should be taken if it is desired to approximate the true average compressive strength <u>within</u> given percentages and probabilities. As an example, it might be specified that 2 per cent and 5 per cent of the average be chosen, at probability levels of 95 and 99 per cent. The equation for computing the number of specimens necessary, is

$$n = \frac{(t)^2\,(v)^2}{(p)^2} \qquad \ldots\ldots.(10)$$

Where n = the number of specimens to be taken
t = a constant corresponding to a probability level. (Values may be obtained from Table V.)
v = The coefficient of variation of the concrete manufacturer, which must be known or estimated.
p = a given percentage of the average strength, that is, say, within some per cent of the average strength.

The number of test specimens to be taken if it is desired to approximate the true average value of the compressive strength within two per cent, and with a probability of 95 per cent when the manufacturers coefficient of variation is 10 per cent, is

$$n = \frac{(1.960)^2(10)^2}{(2)^2} = 96$$

The number of test specimens to be taken if it is desired to approximate the true average value within five per cent, with a probability of 99 per cent and a coefficient of 18 per cent is

$$n = \frac{(2.576)^2(18)^2}{(5)^2} = 86$$

Table VII shows the number of test specimens that should be taken for different probabilities, coefficient, and percentages, of the true average.

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TABLE VII
Estimated Number of Specimens to be Taken

Coefficient of Variation	Probability of 0.95 that Average Compressive Strength Computed from Samples will be Within Given Percentages of True Average Strength		Probability of 0.99 that Average Compressive Strength Computed from Samples will be Within Given Percentages of True Average Strength	
	2%	5%	2%	5%
10	96	15	166	27
12	139	23	239	39
15	216	35	374	60
18	311	50	539	86
20	384	61	666	106

The table shows rather convincingly, that the number of samples that should be taken on a concrete job is closely tied in with the degree of certainty we expect to obtain and also, on the coefficient of variation under which the concrete manufacturer operates. It would certainly seem that a modern ready-mix plant should have no great difficulty in maintaining a coefficient of 12 per cent or better. Under these conditions the number of tests necessary to establish the compressive strength within 5 per cent with a probability of 0.95 is 23. Unless a substantially larger number of tests is made the estimate of true strength will not be greater than illustrated in the preceding sentence. This presupposes of course, that none of the variables in the concrete that can be controlled, are deliberately varied.

Summary

The American Concrete Institute has published approved standards relating to the uniformity of portland cement concrete, and has described the use of statistical measures in making such evaluations.

Other statistical procedures may also be used in the examination of concrete data, and some of them are discussed in this report.

Statistical measures may likewise be employed in evaluating the precision of test methods, and the work of laboratories engaged in concrete testing.

All of the recommended statistical measures are easy to compute and apply, and the committee report notes that "These methods provide tools of considerable value in assessing results of strength tests, and such information is also of value in refining design criteria and specifications."

Statistical Theory

It is not possible in a short report of this kind to engage in any comprehensive discussion of statistical theory. It may be said however, that the basis of the statistical methods used in the study of measurement data is the fact that such data exhibits a definite pattern of variation. This is not a theory, but a fact, and can be easily demonstrated whenever large sets of measurement data are available.

This definite pattern of variation is called a normal distribution, and Figure 1 is a graph of the distribution of the compressive strength values of the first structural concrete project listed in Table II. The data on this project is especially good because it consists of 916 separate tests.

In Figure 1, the abscissas represent the compressive strength in hundreds of pounds per square inch and the ordinates represent the frequencies or number of tests. The rectangles represent the frequencies and compressive strengths within class intervals. Since the width of all the rectangles is the same, their heights are also proportional to their areas. The curve superimposed on the histogram is the normal probability curve with which these data should coincide in theory. It is seen that in this case the data approximate the theoretical curve very closely.

A normal probability curve is completely described by two parameters; the arithmetic mean or average, and the standard deviation, and these universe parameters are validly estimated when we can obtain an unbiased mean and standard deviation from a random sample.

A unit normal probability curve has abscissa values scaled in standard deviations on both sides of the maximum ordinate which is at zero. The total area under the unit normal curve is taken to be 1.0000, and this area represents probability because probability is by definition "relative frequency in the long run."* A probability of 1.0000 represents certainty; a probability of zero represents an impossibility. Probabilities in between are designated as proportions like 0.75, 0.60, 0.99, etc. The fractional areas under the unit curve have all been

*The writer is aware that this definition is a matter of dispute in some quarters.

computed, and are published in all statistical texts as tables of the probability integral. Probabilities are therefore, obtained from tables of the probability integral. Table VIII is an abbreviated table of the probability integral.

It is known that sample means tend toward a normal distribution even though the universe from which they are drawn is not normal. Statistical texts emphasize that in order to utilize this fact, we must have some knowledge of the standard deviation of the universe. An estimate of this value can be made from a sample providing the sample is large enough. With small samples (less than 30), an estimate of the universe standard deviation is not always satisfactory.

In 1908, an English chemist, W. S. Gossett, who wrote under the pseudonym of "Student", computed the distribution for a normal universe for sample values of the statistic

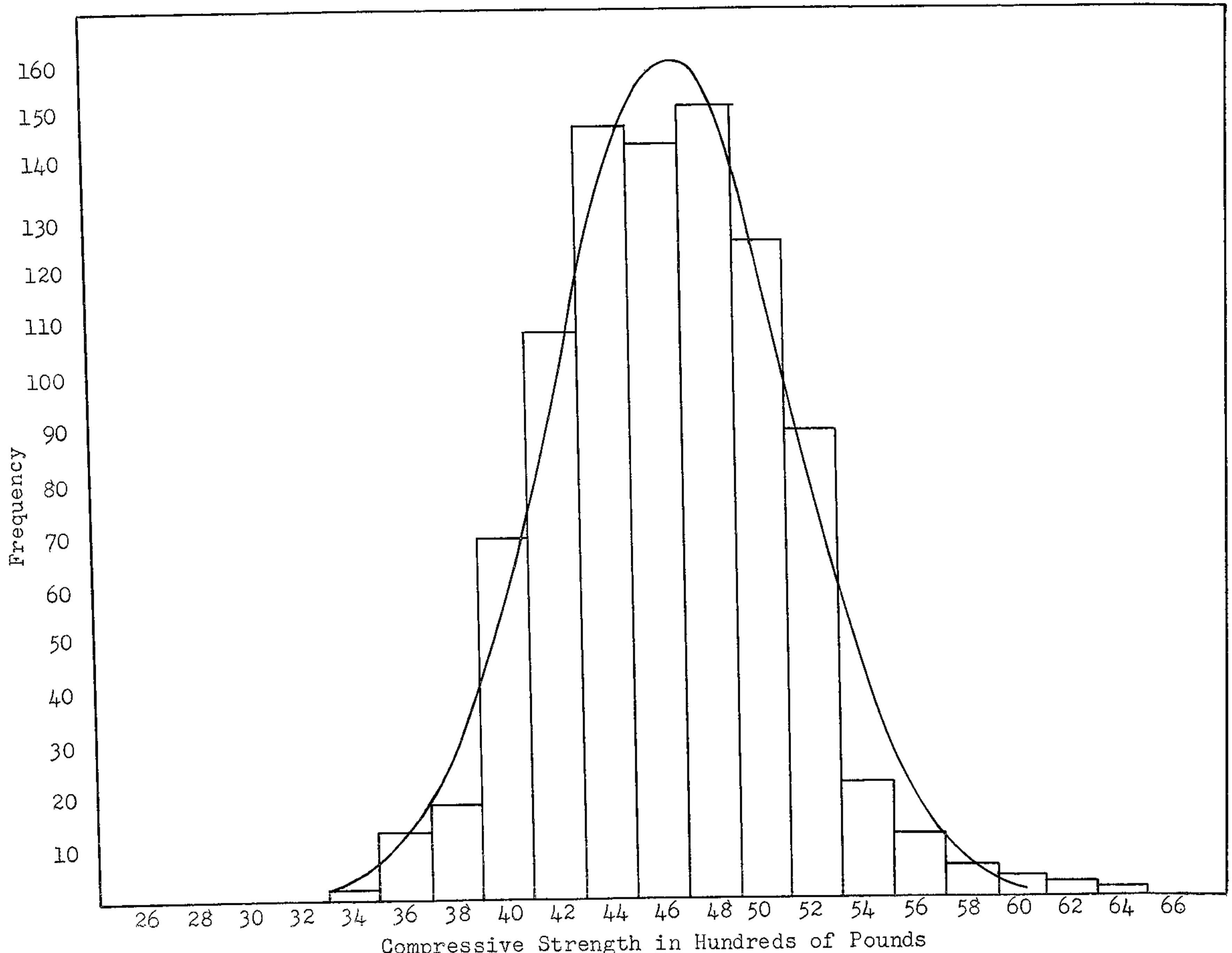
$t = \dfrac{\bar{X} - \bar{X}'}{s/\sqrt{n}}$ where s is an estimate of the universe standard deviation obtained by the use of the equation $s^2 = \dfrac{n}{n-1}\sigma^2$

and $\bar{X}'$ is the universe average. The values in this distribution are those that are given in the well known "student's t" tables, as exemplified by Table V. By using the t tables, the values of $\bar{X}'$ can be approximated for any given level of probability as explained heretofore.

TABLE VIII

The Normal Curve: Ordinates and Areas

Normal Deviate X/σ	Ordinate Y	Prprtn of Area under Segment of Curve Total=1			
		0 to X/σ	-X/σ to X/σ	X/σ to ∞	Outside ± X/σ
(1)	(2)	(3)	(4)	(5)	(6)
.000	.3989	.0000	.0000	.5000	1.0000
.500	.3521	.1915	.3829	.3085	.6171
1.000	.2420	.3413	.6827	.1587	.3173
1.500	.1295	.4332	.8664	.0668	.1336
1.645	.1031	.4500	.9000	.0500	.1000
1.960	.0584	.4750	.9500	.0250	.0500
2.000	.0540	.4772	.9545	.0228	.0455
2.326	.0267	.4900	.9800	.0100	.0200
2.500	.0175	.4938	.9876	.0062	.0124
2.576	.0145	.4950	.9900	.0050	.0100
3.000	.0044	.4987	.9973	.0013	.0027
3.090	.0034	.4990	.9980	.0010	.0020
3.291	.0018	.4995	.9990	.0005	.0010
3.500	.0009	.4998	.9995	.0002	.0005
4.000	.0001	.5000	.9999	.0000	.0001



160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
Frequency
26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66
Compressive Strength in Hundreds of Pounds

Figure 1